

## MODEL TEST FOR VALIDATION OF CALCULATED WAVE-INDUCED EXCITATIONS ON A SHIP IN FOLLOWING AND QUARTERING WAVES

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### Abstract

Semi-captive sea-keeping model tests were performed to measure the heave and pitch motion, wave-induced forces and moments acting on a patrol vessel in following and quartering waves. A numerical method was used to determine the time history of the wave motion around the model ship for every test in form of a wave equation, based upon the measured wave motion by the wave height meter. The same method was also used to extract the first order motion responses and the first order wave-induced forces and moments. Thereby, it has become possible to rationally compare the time histories of the theoretically calculated wave forces and moments with the model measured.

### 1. INTRODUCTION

Numerical modeling is an important part in the study of sea-keeping problems, particularly as a problem is non-linear, such as broaching-to in following waves and slamming impact in heading waves. That is because of the irregular characteristics of ocean waves, and consequently of the ship responses, which make it practically impossible to determine the probabilistic characteristics of the non-linear dynamic problem from model measurement alone. Computer simulation based upon an accurate numerical model has become an efficient tool in this context.

However, any numerical model should be verified by model- or full-scale measurement before being applied for analysis of a practical problem, since there are always some assumptions made under the derivation. For the risk assessment, a numerical model is usually

desired to be as simple as possible, but at the same time adequate and accurate for the studied problem. For establishing such a numerical model, physically relevant assumptions are required and thoroughly proved by model measurement.

Normally, validation of theoretical calculation should at first hand be made for regular waves for a wave-induced problem, even though this kind of wave rarely appears in reality. The reason is that this kind of wave is well defined mathematically. However, it is not a simple task to generate a perfect regular wave with desired parameters in a wave basin. The generated waves are always disturbed in some degrees. Therefore it is important to make proper treatment of the measured data before being used for the purposed validation. Garne and Hua developed in 1999 [1] a method to determine the wave kinematics surrounding the ship model in form of wave equation using the measured wave motion by the wave height

meter. The method is applied to the wave measurements recorded in the sea-keeping experiments on a Ro-Ro vessel. The obtained wave equations accurately repeat the wave gauge signals. The wave equation as a link between a numerical and physical model is verified by comparing measured and simulated heave and pitch motions of the Ro-Ro model in regular and irregular waves. The calculated time series were in good agreement with the measurements.

In this paper, an arrangement of semi-captive model measurement will be described for force and motion measurement of a fast ship in following and quartering waves for validation of theoretically calculated wave-induced excitations on the ship, since these excitations constitute the governing effect on the surfing and broaching problem. The evaluation method by Garne and Hua [1] is shortly described and extended for extraction of the first order motion responses and the first order wave-induced excitations. Then, it becomes possible to compare the time histories of the theoretically calculated wave forces and moments with the model measured.

Three examples are given in section 3. The first one is concerning the wave-induced surge force in following waves and the second concerning the wave-induced surge and sway force and yaw moment in a quartering wave. The third example shows validation of a semi-empirical method for calculation of wave-induced sway force and yaw moment.

## 2. TEST SET-UP AND EVALUATION METHOD

### 2.1 Test set-up

The interesting wave-induced excitations in following and quartering waves are forces at the surge and sway direction respectively and moment at the yaw direction. To measure these

forces and moment for validation of theoretical calculation, the degrees of these motion freedoms have to be restrained. That means that the surge velocity has to be constant, while no sway, roll and yaw motion are allowed. However, the heave and pitch motion (the rotation center of the pitch motion is at the amidships) are allowed to be affected by the wave excitation in order to model the realistic condition.

The semi-captive model test series were carried out in the Maritime Dynamics Laboratory (MDL) at SSPA Sweden. The size of the wave basin is 88 m by 39 m, with wave-maker on two sides. A large multi-motion carriage spans the whole basin. The six-component balance was sited inside the model, which is connected to the carriage through a link, while the model is still allowed to be free to move in the heave and pitch direction respectively after a construction modification. The forces and moments acting on the model were measured,  $F_x$  (surge force),  $F_y$  (sway force),  $F_z$  (heave force),  $M_x$  (roll moment),  $M_y$  (pitch moment) and  $M_z$  (yaw moment), beside the heave and pitch motion.

The sign definitions of the six degrees of motion freedoms are shown in Figure 1. The sign definition of the forces and moments are the same as the six degrees of motion freedoms respectively. The wave height meter is located at the port board of the model and forward the amidships. The measured wave elevation is positive downward as the heave motion.

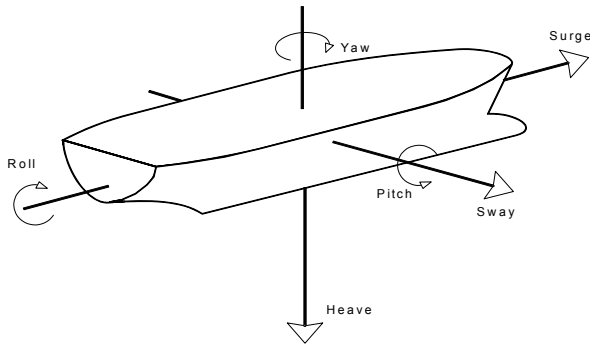


Figure 1 Definition of the six degrees of motion freedoms

The model was built to be the scale 1:18, see Figure 2. The main particulars are shown in Table 1.



Figure 2 The picture of the tested model

Table 1 The main particles of the vessel

Lpp	33.6 m
B	6.2 m
T <sub>f</sub>	1.99 m
T <sub>a</sub>	1.92 m
Displ.	186 m <sup>3</sup>
C <sub>b</sub>	0.457
LCG	-2.08 m
KG	2.93 m

## 2.2 Numerical evaluation method

When analyzing the measured wave-induced responses, it is essential to be able to relate the

measured response signals to the wave motion amidships, i.e. the phase relation. The following is a description how to determine the wave amplitude and the wave phase in the wave equation describing the wave kinematics surrounding the model. The wave equation for the regular wave in the wave basin is expressed as the following

$$\eta(x, t) = a \cdot \cos(k \cdot x - \omega \cdot t) + b \cdot \sin(k \cdot x - \omega \cdot t) \quad (1)$$

The wave height meter is fixed at the model carriage and follows the model movement with the longitudinal coordinate  $x^n(t^n)$ . A time interval is selected for the wave motion with N measured data so that the measured wave data at the different time and space should satisfy the wave equation (1) as the following

$$\begin{bmatrix} \cos(k \cdot x^1 - \omega \cdot t^1) & \sin(k \cdot x^1 - \omega \cdot t^1) \\ \cos(k \cdot x^2 - \omega \cdot t^2) & \sin(k \cdot x^2 - \omega \cdot t^2) \\ \vdots & \vdots \\ \cos(k \cdot x^N - \omega \cdot t^N) & \sin(k \cdot x^N - \omega \cdot t^N) \end{bmatrix} \cdot \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} \eta(x^1, t^1) \\ \eta(x^2, t^2) \\ \vdots \\ \eta(x^N, t^N) \end{Bmatrix} \quad (2)$$

Let

$$[M] = \begin{bmatrix} \cos(k \cdot x^1 - \omega \cdot t^1) & \sin(k \cdot x^1 - \omega \cdot t^1) \\ \cos(k \cdot x^2 - \omega \cdot t^2) & \sin(k \cdot x^2 - \omega \cdot t^2) \\ \vdots & \vdots \\ \cos(k \cdot x^N - \omega \cdot t^N) & \sin(k \cdot x^N - \omega \cdot t^N) \end{bmatrix} \quad (3)$$

so,

$$[M] \cdot \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} \eta(x^1, t^1) \\ \eta(x^2, t^2) \\ \vdots \\ \eta(x^N, t^N) \end{Bmatrix} \quad (4)$$

and

$$\begin{Bmatrix} a \\ b \end{Bmatrix} = ([M]^T \cdot [M])^{-1} \cdot [M]^T \cdot \begin{Bmatrix} \eta(x^1, t^1) \\ \eta(x^2, t^2) \\ \vdots \\ \eta(x^N, t^N) \end{Bmatrix} \quad (5)$$

The coefficients  $a$  and  $b$  in the wave equation are, as seen in the above determined by means of the Least Square Method.

The wave equation can then be used to calculate, for example the wave motion at the ship mass center and other wave kinematics data around the model anywhere and any time. This numerical method can be considered as a filtering procedure. The reconstructed wave is actually the first order harmonic wave component extracted from the measured signal. The difference between the measured and the first order wave component is the sum from the higher order wave components and distortions if the wave is a regular wave. The same numerical methods can also be used to extract the first order components of the wave-induced response such as heave and pitch motion etc. by substituting the measured response signal in steady of the wave signal. In a regular wave, the wave-induced motion responses and excitations can generally be expressed in the following form

$$\begin{aligned} f(t) &= \sum_n f_n(t) = a_0 + \sum_n a_n \cdot \cos(n \cdot \omega \cdot t) + b_n \cdot \sin(n \cdot \omega \cdot t) \\ &= a_0 + \sum_n c_n \cdot \cos(n \cdot \omega \cdot t + \beta_n) \end{aligned} \quad (6)$$

The first order variation is then

$$f_1(t) = a_1 \cdot \cos(\omega \cdot t) + b_1 \cdot \sin(\omega \cdot t) = c_1 \cdot \cos(\omega \cdot t + \beta_1) \quad (7)$$

We can then obtain not only amplitude but also phase in relation to the wave motion.

### 3. SOME VALIDATION CASES

#### 3.1 Importance of wave-induced excitations in surfing and broaching

As know, surfing and broaching can occur to a ship at sufficiently high speed in a following or quartering wave, while located on the down-slope between the wave crest and the wave trough. The ship is subjected to a wave-induced surge force, which can push the ship into acceleration up to the wave speed. Meanwhile, any deviation of the ship forward direction from the incident wave direction can result in a sway force and a yaw moment and consequently sway and yaw velocity and the deviations from the original track. All these secondary motions and velocities cause further increasing sway force and yaw moment. An instable scenario is then formed if the motions cannot be controlled and stopped.

The wave-induced surge and sway force and yaw moment constitute the governing effects on the unstable motion scenario. The basic hydrodynamic mechanisms to these forces and moment are the incident wave potential, the diffraction potential and the hydrodynamic interaction of the incident wave potential with the velocity potential due to the ship forward speed plus viscous effect.

#### 3.2 Calculation method

For theoretical calculation of the wave-induced surge and sway force and yaw moment, we utilize the calculation formulas from [2] by Umeda and Hashimoto. These formulas are derived based upon the strip approach. The wave amplitude is assumed to be small so that the linear relation prevails between the wave-induced excitations and the wave amplitude. Actually, the calculated results are of the first order harmonic excitation with the encounter frequency as variation frequency.

### Surge force:

When a ship runs in a following wave, the resistance can increase and also decrease dependent upon the position the ship is located along the wave. Usually, the resistance in a regular following wave is assumed to be a sum of resistance in calm water  $R_0$  and the wave-induced time-dependent resistance  $\delta R(t) = R_1 \cdot \cos(\omega_e t + \alpha)$ , i.e.

$$R(t) = R_0 + \delta R(t) \quad (8)$$

In the following text we use  $Fx = -\delta R(t)$ . A simple approach to calculate the resistance change in a wave is to adopt the Froude-Krylov hypothesis so that the following expression can be derived

$$Fx = -\rho \cdot g \cdot \zeta_w \cdot k \cdot \cos \beta \cdot \int_L C_1(x) \cdot S(x) \cdot e^{-kd(x)/2} \cdot \sin k(\xi_G + x \cos \beta) \cdot dx \quad (9)$$

where

$$C_1(x) = \frac{\sin(k \sin \beta \cdot B(x)/2)}{k \sin \beta \cdot B(x)/2}.$$

$\xi_G$  is the longitudinal position of the center of gravity from wave trough.

### Sway force:

$$Fy = \rho \cdot g \cdot \zeta_w \cdot k \cdot \sin \beta \cdot \int_L C_1(x) \cdot S(x) \cdot e^{-kd(x)/2} \cdot \sin k(\xi_G + x \cos \beta) \cdot dx + \zeta_w \cdot \omega \cdot \omega_e \cdot \sin \beta \cdot \int_L a_{22}(x) \cdot e^{-kd(x)/2} \cdot \sin k(\xi_G + x \cos \beta) \cdot dx - \zeta_w \cdot \omega \cdot u \cdot \sin \beta \cdot [a_{22}(x) \cdot e^{-kd(x)/2} \cdot \cos k(\xi_G + x \cos \beta)]_{AE}^{FE} \quad (10)$$

### Yaw moment:

$$Mz = \rho \cdot g \cdot \zeta_w \cdot k \cdot \sin \beta \cdot \int_L C_1(x) \cdot S(x) \cdot e^{-kd(x)/2} \cdot x \cdot \sin k(\xi_G + x \cos \beta) \cdot dx + \zeta_w \cdot \omega \cdot \omega_e \cdot \sin \beta \cdot \int_L a_{22}(x) \cdot e^{-kd(x)/2} \cdot x \cdot \sin k(\xi_G + x \cos \beta) \cdot dx + \zeta_w \cdot \omega \cdot u \cdot \sin \beta \cdot \int_L a_{22}(x) \cdot e^{-kd(x)/2} \cdot \cos k(\xi_G + x \cos \beta) \cdot dx - \zeta_w \cdot \omega \cdot u \cdot \sin \beta \cdot [a_{22}(x) \cdot e^{-kd(x)/2} \cdot x \cdot \cos k(\xi_G + x \cos \beta)]_{AE}^{FE} \quad (11)$$

The formula for the wave-induced surge force is derived only taking the effect of the incident wave into account according to the Froude-Krylov hypothesis. The diffraction effect is not considered, nor the interaction between the incident wave and the radiation wave due to the forward speed. Due to this simplification, the calculated wave-induced surge force is speed-independent.

In the calculation of the wave-induced sway force and yaw moment, assumption is also made that the encounter frequency approaches to zero so that the wave memory effect is neglected. The diffraction effect is accounted using the added mass at zero frequency.

### **3.3 Case 1: in following waves**

To validate the calculated first order wave-induced surge force in regular waves, the results of two model tests in following waves are presented here and compared with the calculated according to (9). The first one is in a regular wave with the wavelength equal to the ship length and the second with the wavelength 1.5 times the ship length. The Froude number is 0.3 for the ship speed. The wave equations describing the two regular waves are determined according to the previously described evaluation method.

Figure 3 shows the wave motion re-constructed by means of the wave equation at the location of the wave height meter in comparison with the measured from the first test. The wave equation-given wave motion fits the measured

well in the time-domain. Figure 4 shows the wave motion amidships calculated according to the wave equation. The extracted first order heave and pitch motion together with the measured are shown in the same figure. The first order heave motion is small and about 10% of the wave amplitude since the wavelength is equal to the ship length. The measured heave motion seems to be dominated by the dynamic sinkage and higher order motions. The extracted pitch motion seems fitting the measured well. The extracted first order, the original model measured and calculated wave-induced surge forces are shown in Figure 5 as function of the ship position along the wave, expressed in form of non-dimensional distance from the wave crest. Zero or unit distance means that the amidships is located at the wave crest while 0.5 distance at the wave trough. Obviously, the calculated first order wave-induced surge force according to (9) is overestimated, which is about twice the extracted from the measured result.

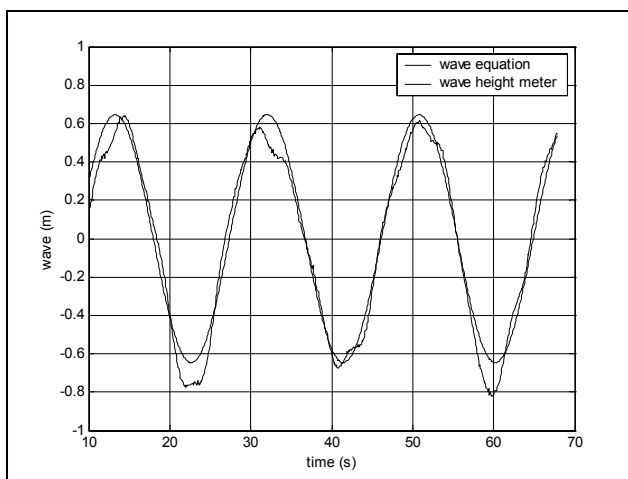


Figure 3 Wave motion given by the wave equation in comparison with the original measured by the wave height meter.

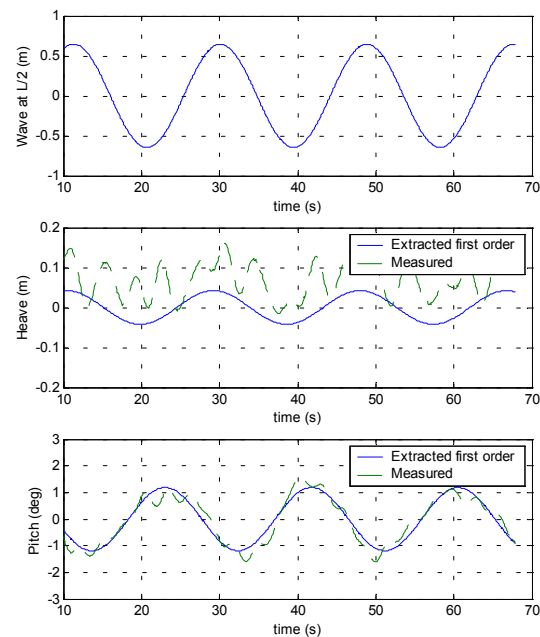


Figure 4 Wave motion at amidships and heave and pitch motion in a following.

The result from the second test is shown in Figure 6, 7 and 8 in the same form as in Figure 3, 4 and 5 respectively. Comparing the time histories of the heave and pitch motion respectively with the wave motion amidships, the heave motion is almost at the same phase as the wave motion, while the pitch motion at a phase difference of about 90 degrees. That means that the heave motion of the vessel follows the wave motion while the pitch motion follows the wave slope. The calculated first order wave-induced surge force in the second test is still greater in comparison with the extracted one from the model measurement, but the discrepancy becomes reduced considerably in comparison with the first test.

Clearly, the Froude-Krylov hypothesis is not sufficiently for theoretical calculation of the first order resistance change. The expression in (9) cannot give acceptable accuracy for a ship in a following regular wave with wavelength near the ship length.

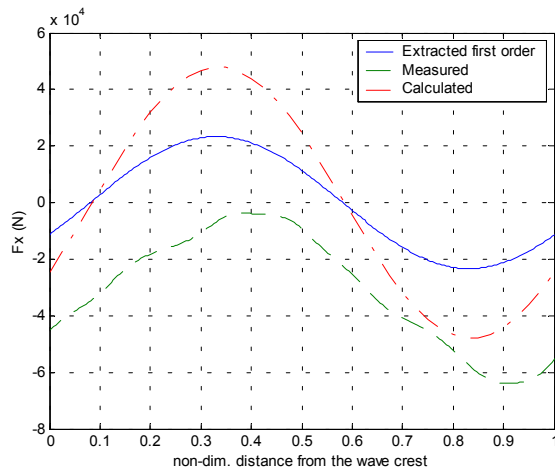


Figure 5 Wave-induced surge force in a follow wave.

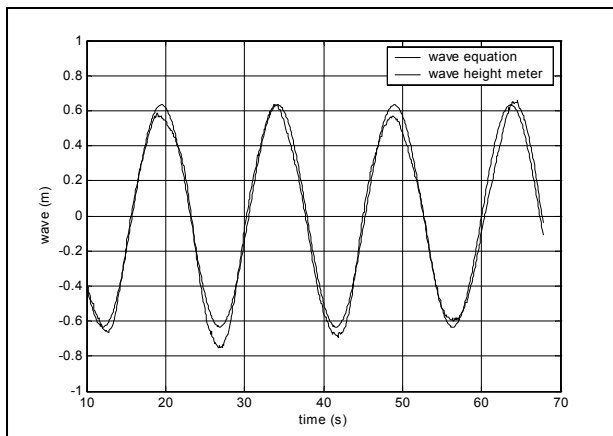


Figure 6 Wave motion given by the wave equation in comparison with the original measured by the wave height meter.

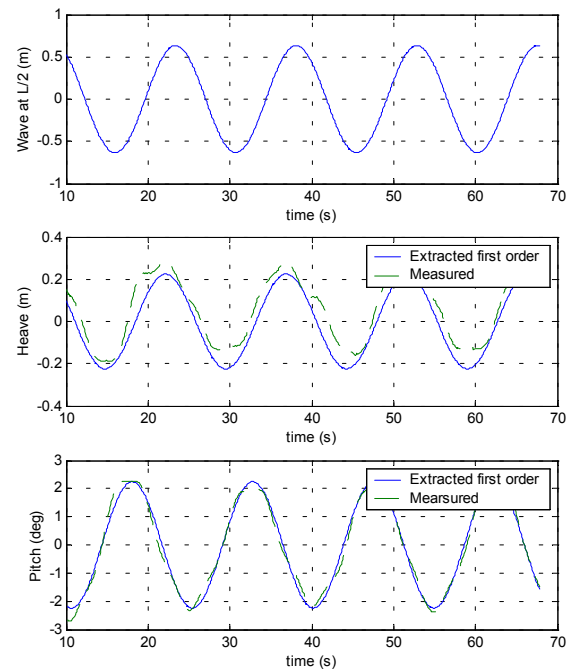


Figure 7 Wave motion at amidships and the heave and pitch motion in a following wave.

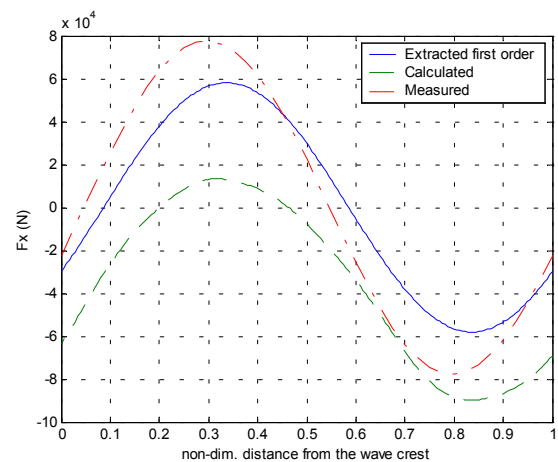


Figure 8 Wave-induced surge force in a follow wave.

### 3.4 Case 2: in quartering waves

Results from two tests of the ship running in regular quartering waves will be presented. The wave incident angle is 30 degrees and the wavelength twice the ship length. Two ship

speeds are considered, expressed in term of Froude number 0.3 and 0.5 respectively.

Figure 9 shows the wave motion at the location of the wave height meter. The wave motion reconstructed from the wave equation fits the measured signal quite well. The extracted first order heave and pitch motion are very near the measured ones and follow the wave and wave slope motion respectively as shown in Figure 10. Thereby, the transfer functions of heave and pitch respectively at this wave condition can be easily determined for both amplitude and phase lag by comparing with the wave motion amidships.

Figure 11 shows the wave-induced surge and sway force and yaw moment respectively in the three sub-figures. The difference between the extracted first order surge force and the measured is almost constant, which is actually quite near the value of the resistance in calm water. The sway force and yaw moment are dominated by their first order components respectively.

Comparison of the calculated wave-induced surge force with the extracted first order one from the measurement is presented in Figure 12. The time histories are expressed in term of non-dimensional distance from the wave crest so that the time variations of the forces are related to the position of the vessel along the wave. The discrepancy is relatively small in comparison with previous two cases in following waves. The explanation may be the longer wavelength. Same comparisons are also made for the wave-induced sway force and yaw moment; see Figure 13 and 14 respectively. Both the calculated wave-induced sway force and yaw moment are underestimated by about 20% relatively to the model measured for the actual run condition. As can be seen, there are also phase discrepancies between the calculated and the model measured ones.

Figures 15 -20 show the same kinds of the results as in Figures 9-14 with the same run condition except the ship speed. The actual speed is 0.5 in Froude number. Due to this speed increase, the vessel is subjected greater dynamic sinkage and trim in comparison with the previous test; see the heave and pitch motion in Figure 16. It has also shown that the relative discrepancies between the calculated wave-induced surge and sway force and yaw moment and the model measured are greater than the results from the previous test with  $F_n=0.3$ . It should also be pointed out that the discrepancies in phase lag increase due to the increased speed.

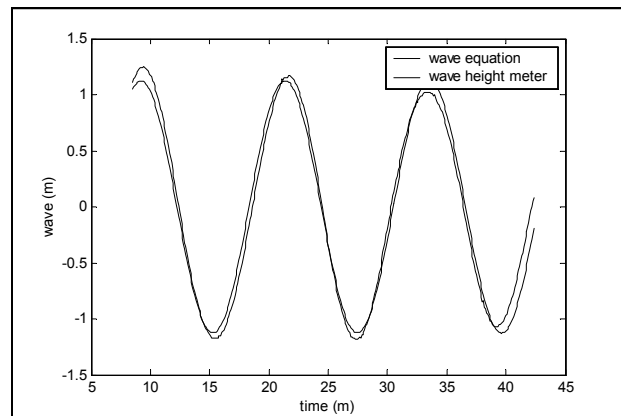


Figure 9 Wave motion given by the wave equation in comparison with the original measured by the wave height meter.



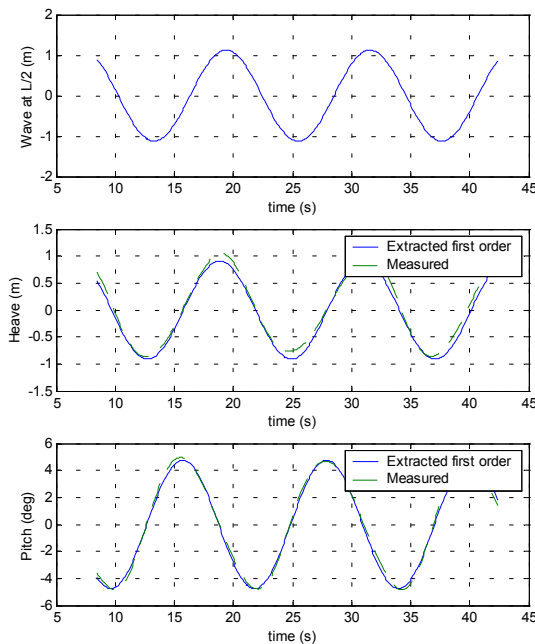


Figure 10 Wave motion at amidships and heave and pitch motion in a following.  $F_n=0.3$ .

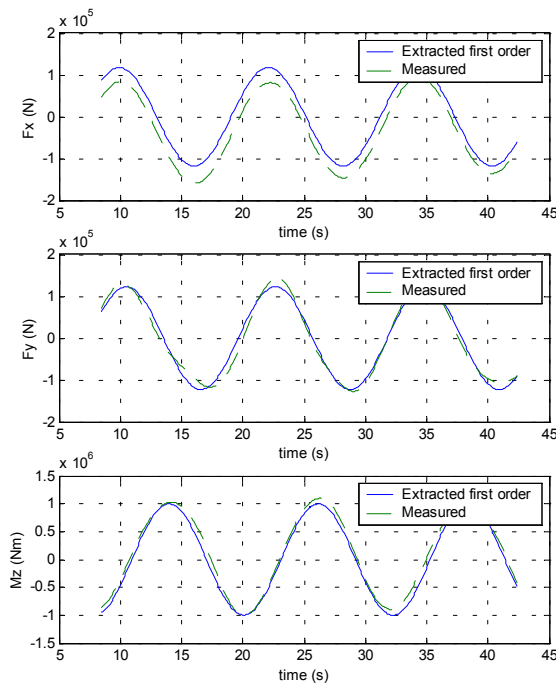


Figure 11 Wave-induced surge and sway force and yaw moment in a quartering wave.  $F_n=0.3$ .

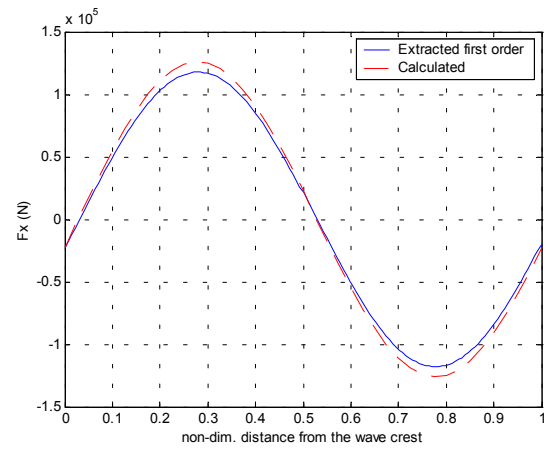


Figure 12 Comparison of the measured and calculated wave-induced surge force in a quartering wave.  $F_n=0.3$ .

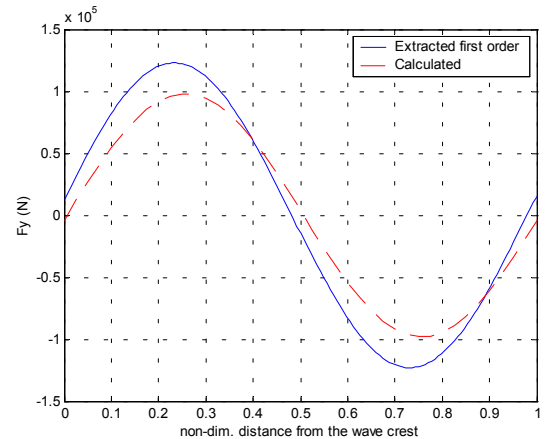


Figure 13 Comparison of the measured and calculated wave-induced sway force in a quartering wave.  $F_n=0.3$ .

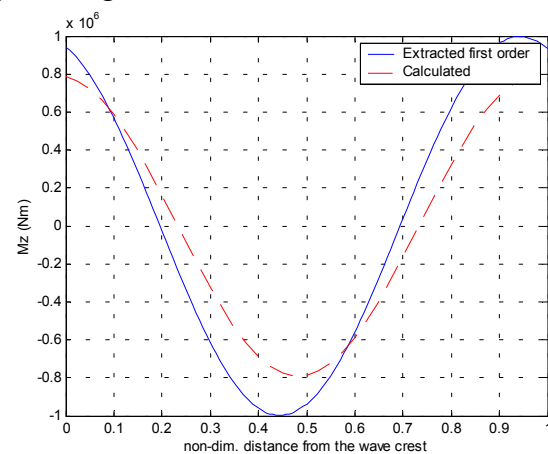


Figure 14 Comparison of the measured and calculated wave-induced yaw moment in a quartering wave.  $F_n=0.3$ .

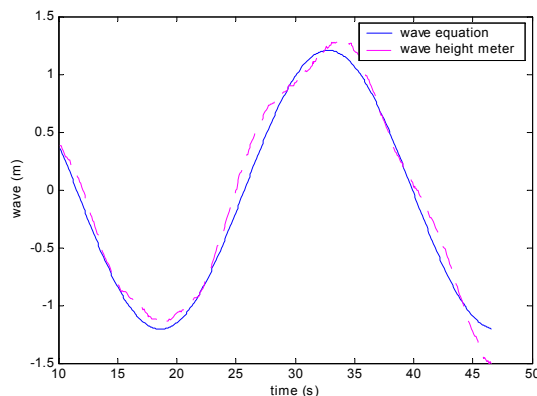


Figure 15 Wave motion given by the wave equation in comparison with the measured by the wave height meter.

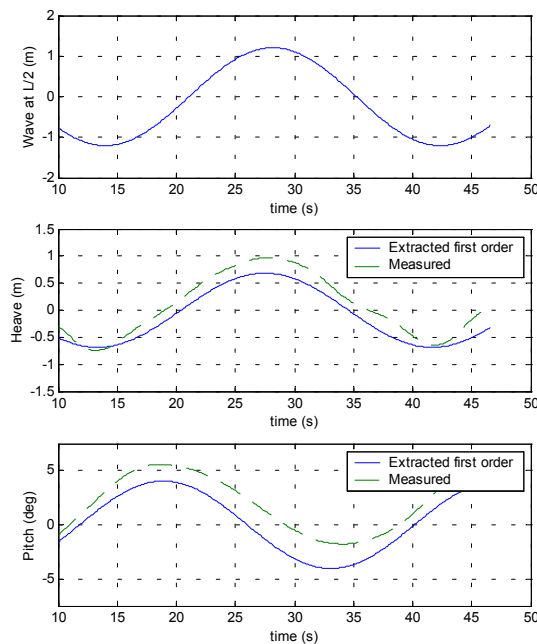


Figure 16 Wave motion at amidships and heave and pitch motion in a quartering wave.  $F_n=0.5$ .

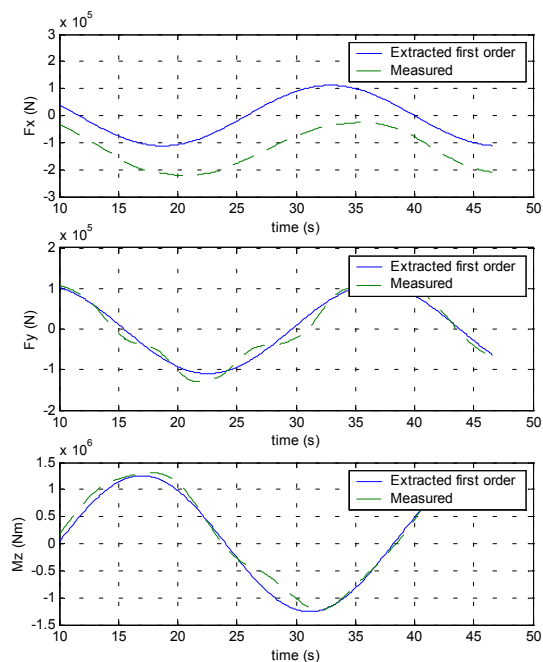


Figure 17 Wave-induced surge, sway force and yaw moment in a quartering wave.  $F_n=0.5$ .

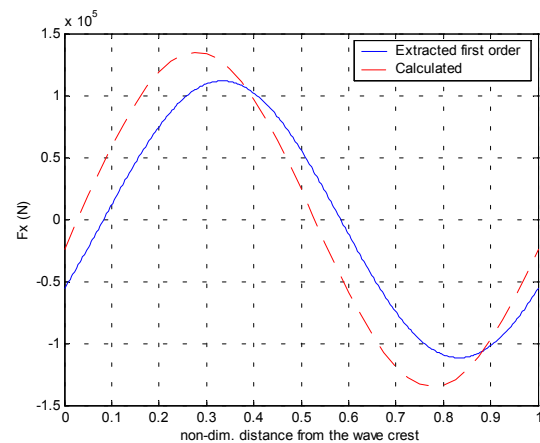


Figure 18 Comparison of the measured and calculated wave-induced surge force in a quartering wave.  $F_n=0.5$ .

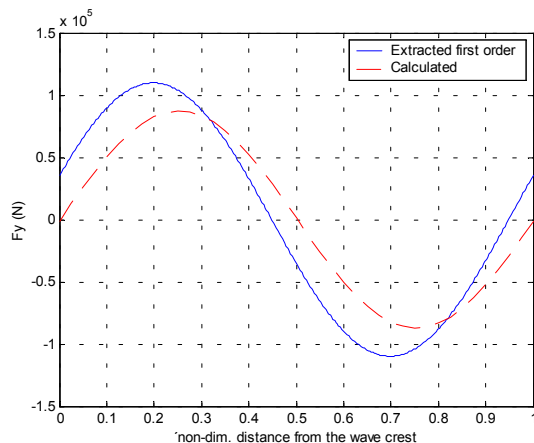


Figure 19 Comparison of the measured and calculated wave-induced sway force in a quartering wave.  $F_n=0.5$ .

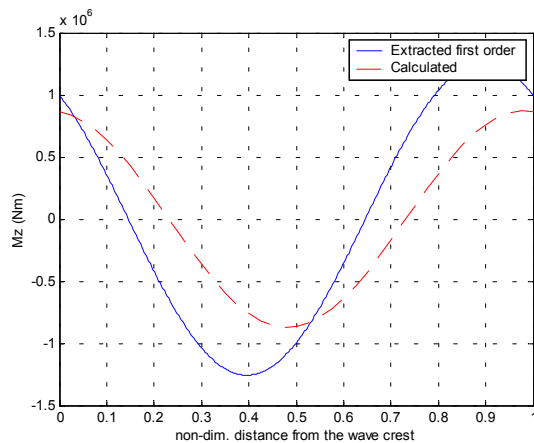


Figure 20 Comparison of the measured and calculated wave-induced yaw moment in a quartering wave.  $F_n=0.5$ .

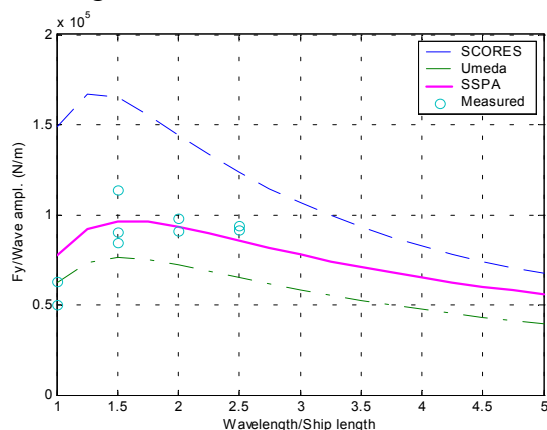


Figure 21 Wave induced sway force on the ship in quartering waves of different wavelengths. The wave incident angle is 30

degrees and the ship speed  $F_n=0.5$ . The legends 'Umeda' and 'SCORES' refer to the methods from [2] and [3] respectively.

### 3.5 Validation of semi-empiric calculation method

Risk assessment in the seakeeping context requires intensive simulation in order to achieve the probabilistic distribution. As known, there are some seakeeping-related hydrodynamic problems, to which available numerical models are insufficiently accurate. As examples, the first two validation examples have shown that the linear strip approach based calculation of the wave-induced excitations in following and quartering waves is not sufficiently accurate. Theoretical improvement needs for these kinds of numerical models, but that will not be easy tasks. Even an advanced numerical model is developed, it means often demand of large computation capacity and consequently it is almost impossible for use in a risk assessment.

A practical approach is to derive semi-empiric calculation method. Figures 21-24 show the comparison of the results of different calculation methods with the model measured wave-induced sway force and yaw moment on the ship in regular quartering waves. The SSPA method in the figures is a semi-empiric method based upon partly the linear strip approach and partly the approach for maneuver force calculation. The result by this method can give satisfactory accuracy for engineering analysis, which is at least available for actual tested ship.

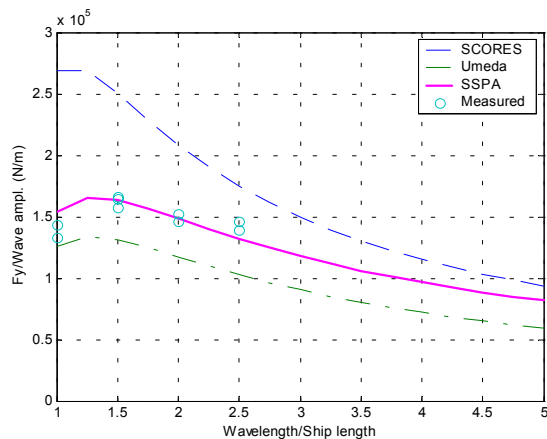


Figure 22 Wave induced sway force on the ship in quartering waves of different wavelengths. The wave incident angle is 45 degrees and the ship speed  $F_n=0.5$ . The legends 'Umeda' and 'SCORES' refer to the methods from [2] and [3] respectively.

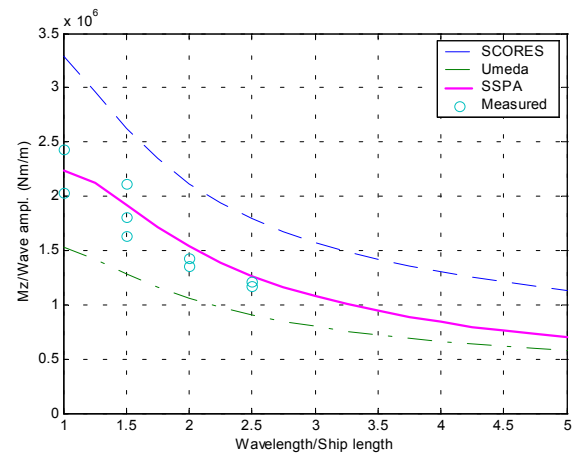


Figure 24 Wave induced yaw moment on the ship in quartering waves of different wavelengths. The wave incident angle is 45 degrees and the ship speed  $F_n=0.5$ . The legends 'Umeda' and 'SCORES' refer to the methods from [2] and [3] respectively.

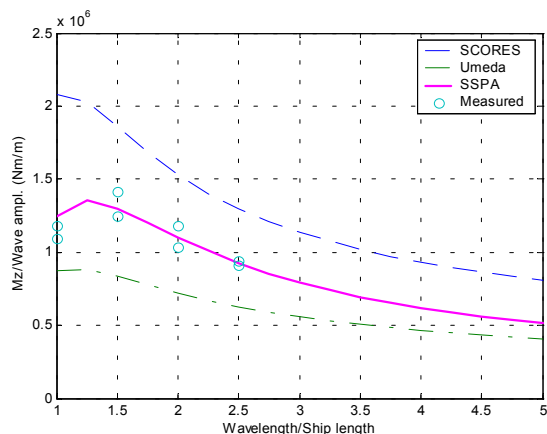


Figure 23 Wave induced yaw moment on the ship in quartering waves of different wavelengths. The wave incident angle is 30 degrees and the ship speed  $F_n=0.5$ . The legends 'Umeda' and 'SCORES' refer to the methods from [2] and [3] respectively.

#### 4. DISCUSSION

As shown in the previous examples, the model measurement results and the evaluated results are satisfactory for validation of theoretical calculation. Theoretically, the basis to the applicability of the evaluation method is that the generated regular wave should be harmonic with constant frequency and amplitude under a run measurement. But, a laboratory-generated wave can be subjected to the change of its kinematic property under the travel from one end to the other of the wave basin, particularly for high wave. The special with the model measurement in a following or quartering wave is that the model encounters limited number of waves under a measurement run, particularly as the encounter frequency approaches to zero. Some of the measured test results have shown that the measured wave elevation became clearly deformed. Normally, the wave crest decreases and the wave period becomes longer. This phenomenon is important for consideration under a validation process.

A numerical wave generator should be established taking the full nonlinear free surface conditions into account. For every run, the corresponding wave should be computer-simulated, which is expected to perform the evaluation and validation procedure easily and reliably.

One difficulty in arranging semi-captive measurement is to avoid the distortion due to the restrain of some degrees of motion freedoms. It requires rigorous check process to insure that all the distortions are insignificant for all the measured signals.

## 5. CONCLUSION

Some results from a series of semi-captive model tests for a patrol vessel in following and quartering waves are presented in this paper. A numerical method was used in the analysis of the measured results to determine the time history of the wave motion around the model ship for every test in form of a wave equation. The same method was also used to extract the first order wave-induced forces and moments, which together constitute the decisive effect in the instability mechanism of a ship in following and quartering waves. Theoretical calculations of the wave-induced first order surge and sway force and yaw moment were conducted and the results were compared with the model measured.

The comparison and validation examples have demonstrated that the generated harmonic waves are of high quality and in combination with the described evaluation method are appropriate for validation of theoretical calculation. Certainly, the presented test technique and evaluation method can also be applied for the following sea-keeping problems:

- Hydrodynamic interaction between the different hydrodynamic mechanisms under a broaching scenario
- Hydrodynamic interaction between two or several regular wave components and its effect on the wave excitations under a broaching scenario
- Stability loss in following waves
- Parametrically excited roll motion in following waves
- Water ingress in waves due to hull damage
- Slamming impact in bow and heading waves
- Effect of wave and wave-induced ship motions on the rudder force

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## 7. REFERENCES

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